OKHEP-00-05

EIKONAL SCATTERING OF MONOPOLES AND DYONS IN DUAL QED

LEONARD GAMBERG[a,b] and KIMBALL A. MILTON[c]
Department of Physics and Astronomy, University of Oklahoma
Norman, OK 73019

The quantum field theory of electron-point magnetic monopole interactions and dyon-dyon interactions, based on the string-dependent "nonlocal" action of Dirac and Schwinger is developed. We demonstrate that a nonperturbative quantum field theoretic formulation can be constructed resulting in a string *independent* cross section for monopole-electron and dyon-dyon scattering. Such calculations can be done only by using nonperturbative approximations such as the eikonal and not by some mutilation of lowest-order perturbation theory.

1 Introduction

The topic of magnetic charge has received enormous attention since Dirac [1] demonstrated its existence was consistent with quantum mechanics provided the quantization condition (in rationalized units) $eg/4\pi = N/2$ is satisfied. Here e and g are the strength of electric and magnetic charges, respectively, and N denotes an integer. In the case of dyons, particles containing both magnetic and electric charge, the Schwinger generalization [2,3]

$$\frac{e_a g_b - e_b g_a}{4\pi} = \left\{ \begin{array}{l} \frac{N}{2}\,, \text{unsymmetric} \\ N\,, \text{symmetric} \end{array} \right\}, \tag{1}$$

is invoked. ("Symmetric" and "unsymmetric" refer to the presence or absence of dual symmetry in the solutions of Maxwell's equations.)

With the advent of non-Abelian theories, classical composite monopole solutions were (theoretically) discovered. [4] Their mass would be of the order of the relevant gauge-symmetry breaking scale, which for grand unified theories (GUT) is of order 10^{16} GeV or higher. However, there are models where the electroweak symmetry breaking can give rise to monopoles of mass ~ 10 TeV. [5] Yet, even the latter are not accessible to accelerator experiments, so limits on heavy monopoles depend either on cosmological considerations, or detection of cosmologically produced (relic) monopoles impinging upon the earth or moon. [6]

[a] Invited talk given at "5$^{\text{th}}$ Workshop on QCD", Villefranche-sur-Mer, France, 3-7 Jan. 2000.
[b] E-mail: gamberg@mail.nhn.ou.edu
[c] E-mail: milton@mail.nhn.ou.edu

However, *a priori*, there is no reason that Dirac/Schwinger monopoles or dyons of arbitrary mass might not exist. In this respect, it is important to set limits below the 1 TeV scale in direct accelerator based experiments. [d]

It is envisaged that if monopoles are sufficiently light, they would be produced by a Drell-Yan type of process occurring in $p\overline{p}$ collisions at the Tevatron. The difficulty is to derive a reasonable estimate of the elementary process $q\overline{q} \to \gamma^* \to M\overline{M}$, where q stands for quark and M for magnetic monopole. Attempts to incorporate monopoles consistently into relativistic quantum field theory have met with mixed success. Weinberg, and soon thereafter Rabl, [10] demonstrated that the charge-monopole scattering amplitude, calculated in the one-photon-exchange approximation, is a function of the *Dirac string singularity*. Making matters worse, the value of the vertex coupling implied by $\alpha_g = g^2/4\pi \approx 34N^2$, calls into question any approach based on a badly divergent perturbative expansion in α_g. Although the early efforts using a Feynman-rule perturbation theory resulted in string-dependent cross–sections, subsequently *ad hoc* assumptions were invoked to render the resulting cross-sections string independent (see Ref. [11] for details). In contrast, studying the *formal* behavior of Green's functions in the relativistic quantum field theory of electrons and monopoles, both Schwinger[2,3] and Brandt *et al.*[12] demonstrated Lorentz-string and gauge invariance.

However, with the exception of one instance[13] such demonstrations have been conspicuously absent at the phenomenological level.[e] This deficiency stems from the fact that in most phenomenological treatments of charge-monopole processes the "string independence" of the quantum field theory and the strength of the coupling are treated as separate issues. In fact, these two points are intimately related. The lesson to be learned from the formal and non-relativistic demonstrations of Lorentz and string invariance is this: Because the quantization condition is intimately tied to the demonstration of Lorentz invariance, the latter can only be demonstrated using a method which does not rely on perturbation theory (see[11] for further discussion).

In view of the necessity of establishing a reliable estimate for monopole production in accelerators in order to be able to set bounds on monopole masses, it is important to put the theory of dual quantum electrodynamics (dual QED) on a firmer foundation. With that in mind we present our results. [11]

---

[d]Such an experiment is currently in progress at the University of Oklahoma,[7] where we have set limits on *direct* monopole production at Fermilab up to several hundred GeV. This is an improvement over previous limits.[8] See also Ref.[9] for critique of theories underlying indirect searches.

[e]This is surprising because one expects that the invariant non-relativistic scattering result (see Ref. [15] and references therein) corresponds in a certain kinematic regime to a infinite summation of a particular *subclass* of Feynman diagrams.

2 Dual Electrodynamics

For a spin $\frac{1}{2}$ monopole, a minimal generalization of the QED action [3,11] for charge-monopole interactions expressed in terms of the vector potential A_μ and field strength tensor $F_{\mu\nu}$ (*i.e.* , in a first-order formalism) is

$$W = \int (dx) \Big\{ -\frac{1}{2} F^{\mu\nu}(x) \left(\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\right) + \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x)$$
$$+ \bar{\psi}(x) \left(i\gamma\partial + e\gamma A(x) - m_\psi\right) \psi(x) + \bar{\chi}(x) \left(i\gamma\partial + g\gamma B(x) - m_\chi\right) \chi(x) \Big\}, \quad (2)$$

where it is assumed that the electrically and magnetically charged particles are spin 1/2. The resulting Maxwell's equations, which imply the dual conservation of electric and magnetic currents, j_μ and ${}^*j_\mu$, necessitates the introduction of the Dirac string singularity. The Dirac string function satisfies the differential equation $\partial_\mu f^\mu(x) = \delta(x)$, which has the formal solution $f^\mu(x) = n^\mu (n\cdot\partial)^{-1} \delta(x)$, where n^μ is an arbitrary vector.[f] The field equations resulting from $\delta W = 0$ are $\partial_\nu F^{\mu\nu} = j^\mu$ and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + {}^*G_{\mu\nu}, \quad (3)$$

where

$$G_{\mu\nu}(x) = \int (dy) \left(f_\mu(x-y) {}^*j_\nu(y) - f_\nu(x-y) {}^*j_\mu(y)\right). \quad (4)$$

The auxiliary dual field B_μ is defined as a functional of field-strength and depends on the string function,

$$B_\mu(x) = -\int (dy) f^\nu(x-y) \; {}^*F_{\mu\nu}(y). \quad (5)$$

Of course, the monopole field satisfies the Dirac equation

$$\left(i\gamma\partial + g\gamma B(x) - m_g\right) \chi(x) = 0\,. \quad (6)$$

From Eq. (5) we find that B_μ satisfies $\int (dx') f^\mu(x-x') B_\mu(x') = 0$, which is a special case of a gauge-fixed vector field defined in terms of the field strength through an *inversion* formula, Eq. (5). Similarly, we are at liberty to restrict the vector potential, A_μ to a hypersurface in field space embodied in the inversion formula

$$A_\mu(x) = -\int (dy) f^\nu(x-y) \; F_{\mu\nu}(y)\,, \quad (7)$$

[f] Here we have chosen the string to satisfy the oddness condition (this is the "symmetric" solution) $f^\mu(x) = -f^\mu(-x)$, corresponding to Schwinger's integer quantization condition.[15]

which we denote as *string-gauge*, $\int(dx')f^\mu(x-x')A_\mu(x')=0$. The photon kernel derived from the corresponding gauge fixed action now possesses an inverse

$$D_{\mu\nu}(x)=\left[g_{\mu\nu}-\frac{n_\mu\partial_\nu+n_\nu\partial_\mu}{(n\cdot\partial)}+\left(1-\frac{1}{\kappa}\frac{(n\cdot\partial)^2\partial^2}{n^2}\right)\frac{n^2\partial_\mu\partial_\nu}{(n\cdot\partial)^2}\right]D_+(x)\,, \tag{8}$$

where $D_+(x)$ is the massless scalar propagator,

$$D_+(x)=\frac{1}{-\partial^2-i\epsilon}\delta(x)\,, \tag{9}$$

which enables us to write an integral equation, expressing the vector potential, A_μ (B_μ) in terms of the electric (magnetic) and magnetic (electric) currents. We generalize these classical integral equations to one point Green functions in obtaining the generating function for Green's functions in dual QED.

3 Quantization of Dual QED: Schwinger-Dyson Equations

Using a path integral formulation to quantize the string-dependent action is by no means straightforward. In order to unambiguously develop the generating functional for physical processes we make use of Schwinger's quantum action principle,[16] where we write the vacuum persistence amplitude for Green functions in the presence of external sources, $Z(\mathcal{J})=\langle 0_+\mid 0_-\rangle^{\mathcal{J}}$ for the charge-monopole system. That is, under an arbitrary variation,

$$\delta\langle 0_+\mid 0_-\rangle^{\mathcal{J}}=i\langle 0_+\,|\delta W(\mathcal{J})|\,0_-\rangle^{\mathcal{J}}, \tag{10}$$

where $W(\mathcal{J})$ is the action given in Eq. (2) externally driven by the sources, $\mathcal{J}$, which for the present case are given by the set of terms

$$W(\mathcal{J})=W+\int(dx)\left\{J^\mu A_\mu+{}^*J^\mu B_\mu+\bar\eta\psi+\bar\psi\eta+\bar\xi\chi+\bar\chi\xi\right\}. \tag{11}$$

Given the one-point functions ($\mathcal{O}_\mu$ is the field conjugate to the source $\mathcal{J}^\mu$)

$$\frac{\delta\log Z(\mathcal{J})}{i\delta\mathcal{J}^\mu(x)}=\frac{\langle 0_+|\mathcal{O}_\mu(x)|0_-\rangle^{\mathcal{J}}}{\langle 0_+\mid 0_-\rangle^{\mathcal{J}}}, \tag{12}$$

we solve the corresponding coupled Schwinger-Dyson equations for the vacuum amplitude[11], subject to the gauge conditions

$$\int(dx')f^\nu(x-x')\frac{\delta\langle 0_+|0_-\rangle_0^{\mathcal{J}}}{\delta J^\nu(x')}=0,\quad \int(dx')f^\mu(x-x')\frac{\delta\langle 0_+|0_-\rangle^{\mathcal{J}}}{\delta^*J_\mu(x')}=0. \tag{13}$$

Since any expansion in α_g or eg is not practically useful we recast the solution into a functional form better suited for a nonperturbative calculation of the four-point Green's function. [11] For dyons, the different species of which are labeled by the index a this is

$$\begin{aligned} Z(\mathcal{J}) &= \exp\left\{\frac{i}{2}\int (dx)(dx')\mathcal{K}^\mu(x)\mathcal{D}_{\mu\nu}\left(x-x'\right)\mathcal{K}^\nu\left(x'\right)\right\} \\ &\times \exp\left\{\frac{i}{2}\int (dx)(dx')\frac{\delta}{\delta\bar{\mathcal{A}}_\mu(x)}\mathcal{D}_{\mu\nu}(x-x')\frac{\delta}{\delta\bar{\mathcal{A}}_\nu(x')}\right\} \\ &\times \exp\left\{i\sum_a\int (dx)(dx')\bar{\zeta}_a\left(x\right)G_a(x,x'|\bar{\mathcal{A}}_a)\zeta_a\left(x'\right)\right\} \\ &\times \exp\left\{-\sum_a\int_0^1 dq\,\mathrm{Tr}\gamma\bar{\mathcal{A}}_a G_a(x,x|q\bar{\mathcal{A}}_a)\right\}, \end{aligned} \tag{14}$$

where $\mathcal{A}_a = e_a A + g_a B$, ζ_a is the source for the dyon of species a, and a matrix notation is adopted,

$$\mathcal{K}^\mu(x) = \begin{pmatrix} J(x) \\ {}^*J(x)\end{pmatrix}, \qquad \frac{\delta}{\delta\bar{\mathcal{A}}_\mu(x)} = \begin{pmatrix} \delta/\delta\bar{A}_\mu(x) \\ \delta/\delta\bar{B}_\mu(x)\end{pmatrix},$$

$$\mathcal{D}_{\mu\nu}\left(x-x'\right) = \begin{pmatrix} D_{\mu\nu}\left(x-x'\right) & -\tilde{D}_{\mu\nu}\left(x-x'\right) \\ \tilde{D}_{\mu\nu}\left(x-x'\right) & D_{\mu\nu}\left(x-x'\right)\end{pmatrix}. \tag{15}$$

We use the shorthand notation for the "dual propagator" that couples magnetic to electric charge

$$\tilde{D}_{\mu\nu}\left(x-x'\right) = \epsilon_{\mu\nu\sigma\tau}\int (dx'')D_+\left(x-x''\right)\partial''^\sigma f^\tau\left(x''-x'\right). \tag{16}$$

The two-point fermion Green's functions in the background of the stationary photon field $\bar{A}, \bar{B}$ are given by

$$G(x,x'|\bar{\mathcal{A}}) = \langle x|(\gamma p + m - \bar{\mathcal{A}})^{-1}|x'\rangle. \tag{17}$$

4 Eikonal Approximation for Dyon-Dyon and Charge-Monopole Scattering

To calculate the dyon-dyon scattering cross section we obtain the four–point Green's function for this process from Eq. (14),

$$G\left(x_1,y_1;x_2,y_2\right) = \frac{\delta}{i\delta\bar{\zeta}_1(x_1)}\frac{\delta}{i\delta\zeta_1(y_1)}\frac{\delta}{i\delta\bar{\zeta}_2(x_2)}\frac{\delta}{i\delta\zeta_2(y_2)}Z(\mathcal{J})\Big|_{\mathcal{J}=0}. \tag{18}$$

The subscripts on the sources refer to the two different dyons. As a first step in analyzing the string dependence of the scattering amplitudes, we study high-energy forward scattering processes where *soft* photon exchanges dominate. Diagrammatically, in this kinematic regime we restrict attention to that subclass in which there are no closed fermion loops and the photons are exchanged between fermions.[17] This amounts to quenched-ladder approximation where the linkage operators, Ł, connect two fermion propagators via photon exchange. We can read this off from Eq. (14):

$$\mathrm{e}^{\text{Ł}_{12}} = \exp\left\{ i\int (dx)(dx')\frac{\delta}{\delta\bar{\mathcal{A}}_1^{\mu}(x)}\mathcal{D}^{\mu\nu}\left(x-x'\right)\frac{\delta}{\delta\bar{\mathcal{A}}_2^{\nu}(x')}\right\}, \tag{19}$$

so Eq. (18) takes the form

$$G\left(x_1,y_1;x_2,y_2\right) = -\mathrm{e}^{\text{Ł}_{12}} G_1(x_1,y_1|\bar{\mathcal{A}}_1)G_2(x_2,y_2|\bar{\mathcal{A}}_2)\Big|_{\bar{A}=\bar{B}=0}, \tag{20}$$

where we express the two-point function using the proper-time parameter representation of an ordered exponential. The soft, nonperturbative effects of the interaction between electric and magnetic charges dominate in the region where the momentum exchanged by the photons is small compared to the center of mass energy $s=-(p_1+p_2)^2$, *i.e.* $t/s \ll 1$. This amounts to the Bloch-Nordsieck[18] or *eikonal approximation.*[19,20] This approximation substantially simplifies evaluating the path-ordered exponentials in Eq. (20). They are now exponentials of linear functionals of the gauge field.

4.1 High Energy Scattering Cross Section

Using the identity,

$$\mathrm{e}^{\text{Ł}} = 1 + \int_0^1 da\, \mathrm{e}^{a\text{Ł}}\text{Ł} \tag{21}$$

one obtains to the following form of the four-point Green function, Eqs. (20),

$$\begin{aligned} G(x_1,y_1;x_2,y_2) = &\int_0^1 da \int (dz_1)(dz_2)\overline{\mathcal{D}}_{\mu\nu}(z_1-z_2)\mathrm{e}^{a\text{Ł}_{12}} \\ &\times G_1(x_1,z_1|\bar{\mathcal{A}}_1)\gamma_\mu G_1(z_1,y_1|\bar{\mathcal{A}}_1)G_2(x_2,z_2|\bar{\mathcal{A}}_2)\gamma^\nu G_2(z_2,y_2|\bar{\mathcal{A}}_2)\Big|_{\bar{A}=\bar{B}=0}, \end{aligned} \tag{22}$$

where, $\overline{\mathcal{D}}_{\mu\nu}(x)$ represents the combination of propagators,

$$\overline{\mathcal{D}}_{\mu\nu}(z_1-z_2) = \mathsf{q}_1\cdot\mathsf{q}_2 D_{\mu\nu}(z_1-z_2) - \mathsf{q}_1\times\mathsf{q}_2\tilde{D}_{\mu\nu}(z_1-z_2), \tag{23}$$

and the charge combinations invariant under duality transformations are

$$\mathsf{q}_1 \cdot \mathsf{q}_2 = \; e_1 e_2 \; + \; g_1 g_2 \quad \text{and} \quad \mathsf{q}_1 {\times} \mathsf{q}_2 = e_1 g_2 - g_1 e_2 \; . \tag{24}$$

Choosing a space like string, $n^\mu = (0, \mathbf{n})$ and the incoming momenta to be in the z direction, in the center of momentum frame, the Møller amplitude, $M(s,t)$ is given by

$$M(s,t) = \frac{-i}{2\pi} \int_0^1 da \int d^2\mathbf{x}\, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}} \bar{u}(p_1')\gamma^\mu u(p_1) \bar{u}(p_2')\gamma^\nu u(p_2)\, \mathrm{e}^{ia\Phi_n(s,t;x)}$$
$$\left\{ g_{\mu\nu} \mathsf{q}_1 \cdot \mathsf{q}_2 K_0\left(\mu\left|\mathbf{x}\right|\right) - \epsilon_{\mu\nu\sigma\tau} \mathsf{q}_1 {\times} \mathsf{q}_2 n^\tau \frac{\partial}{\partial n_\sigma} \frac{1}{2} \int \frac{d\mathrm{t}}{\mathrm{t}} K_0\left(\mu\left|(\mathbf{x} + \mathrm{t}\mathbf{n})\right|\right) \right\}, \tag{25}$$

where in this kinematic regime, the eikonal phase is (μ is the photon mass; $\tilde{\mu} = \mathrm{e}^\gamma \mu/2$ and γ is Euler's constant)

$$\Phi_n\left(s,t;x\right) = \frac{1}{2\pi} \left\{ \mathsf{q}_1 \cdot \mathsf{q}_2 \ln\left(\tilde{\mu}\left|\mathbf{x}\right|\right) - \mathsf{q}_1 {\times} \mathsf{q}_2 \arctan\left[\frac{\hat{\mathbf{n}} \cdot \mathbf{x}}{\hat{\mathbf{z}} \cdot (\hat{\mathbf{n}} \times \mathbf{x})} \right] \right\}. \tag{26}$$

First, we calculate the helicity amplitudes in the high-energy limit, $p^0 \gg m$. In performing the integral over the impact parameter care must be taken since the arctangent function is discontinuous when the xy component of $\hat{\mathbf{n}}$ and $\mathbf{x}$ lie in the same direction. However, requiring that the eikonal phase factor $e^{i\Phi_n}$ be continuous, necessarily leads to the Schwinger quantization condition (1): $\mathsf{q}_1 {\times} \mathsf{q}_2 = 4N\pi$. Now using the integral form for the Bessel function of order ν

$$i^\nu J_\nu(t) = \int_0^{2\pi} \frac{d\phi}{2\pi} \mathrm{e}^{i(t\cos\phi - \nu\phi)} \, , \tag{27}$$

we find the dyon-dyon scattering amplitude to be

$$M(s,t) = \frac{s}{M_1 M_2} \frac{2\pi}{q^2} (N - i\tilde{\alpha}) \mathrm{e}^{i2N\psi} \left(\frac{4\tilde{\mu}^2}{q^2} \right)^{i\tilde{\alpha}} \frac{\Gamma\left(1 + N + i\tilde{\alpha}\right)}{\Gamma\left(1 + N - i\tilde{\alpha}\right)} \, , \tag{28}$$

where $\tilde{\alpha} = \mathsf{q}_1 \cdot \mathsf{q}_2 / 4\pi$, and ψ is the angle between $\mathbf{q}$ and $\mathbf{n}$.

This result is almost identical in structure to the non-relativistic form of the scattering amplitude for the Coulomb potential, which result is recovered by setting $N = 0$ (see, for example, Ref.[20].) Following the standard convention we calculate the spin-averaged cross section for dyon-dyon scattering in the high energy limit,

$$\frac{d\sigma}{dt} = \frac{\left(\mathsf{q}_1 \cdot \mathsf{q}_2\right)^2 + \left(\mathsf{q}_1 {\times} \mathsf{q}_2\right)^2}{4\pi t^2}. \tag{29}$$

For the case of charge-monopole scattering $e_1 = g_2 = 0$, this result, coincides with that found by Urrutia [g] which is also string independent as a consequence of the quantization condition.

5 Corrections

To go beyond the regime of soft or infrared photon exchange requires a detailed analysis of factorization of soft and hard contributions and their correlations. Such effects have been widely studied in the context of phenomenologically based hadronic interacting field theories [21] and more recently in the context of diffractive scattering [17] in QCD and in the world-line formalism. [22,23]

As a reasonable first step we impose corrections on the eikonal amplitude by relaxing the "high energy" approximation on the spinors in Eq. (25) while assuming that the soft contributions are dominated by the eikonal phase. [24] This result, which we expect to be a better and better approximation in the high energy limit (for a given t), *i.e.* $t/s \to 0$, obeys the expected scaling behavior, for electron monopole scattering

$$s^2 \frac{d\sigma}{dt} = \frac{(eg)^2}{4\pi} \frac{1}{t^2} (s+t)^2 \Rightarrow f\left(\frac{t}{s}\right). \tag{30}$$

Assuming that we have extended the kinematic range of this result beyond the low t^2 limit, we can consider using the analytic properties of the scattering amplitude to calculate the Drell-Yan production amplitude in the t-channel. Detail will be presented in future publication.

6 Conclusions

We have given a complete formulation, in modern quantum field theoretic language, of an interacting electron-monopole and dyon-dyon systems. The challenge is to apply these equations to the calculation of monopole and dyon processes. Perturbation theory is useless, not only because of the strength of the coupling, but also because the graphs are fatally string- (or gauge-) dependent. The most obvious nonperturbative technique for transcending these limitations in scattering processes lies in the high energy regime where the eikonal approximation is applicable; in that limit, our formalism generalizes an early lowest-order result of Urrutia and charts the way to include systematic corrections. More problematic is the treatment of monopole production processes.

[g] Utilizing Schwinger's functional source theory [14] in the context of a zeroth order eikonal approximation Urrutia [13] demonstrated string independence of the charge-monopole scattering cross section, although in his treatment the currents are approximated by those of classical point particles.

We will apply the techniques and results found here to the Drell-Yan production of monopole-antimonopole processes, and obtain phenomenologically relevant estimates for the accelerator production of monopole-antimonopole pairs.

In addition we have also detailed how the Dirac string dependence disappears from physical quantities. It is by no means a result of string averaging or a result of dropping string-dependent terms (see Ref. [11] for details); but rather, a result of summing the soft contributions to the dyon-dyon or electron-monopole process. At the level of the eikonal approximation and its corrections one might suspect the occurrence of a factorization of hard string-independent and soft string-dependent contributions in a manner similar to that argued recently in strong-coupling QCD. There is reason to believe that inclusion of soft-hard correlations in the scattering will not spoil this consistency.

Acknowledgments

L.G. would like to thank Herb Fried and the organizers of "Fifth Workshop on QCD" for the invitation to present this work. This work is supported in part by the Department of Energy.

References

1. P. A. M. Dirac, *Proc. R. Soc.* (London) **A 133**, 60 (1931).
2. J. Schwinger, *Phys. Rev.* **173**, 1536 (1968).
3. J. Schwinger, *Phys. Rev.* D **12** , 3105 (1975).
4. A. M. Polyakov, JETP Lett. **20**, 194 (1974); Y. Nambu, *Phys. Rev.* D **10**, 4262 (1974); G. 't Hooft, *Nucl. Phys.* **B79**, 276 (1974).
5. J. Preskill, *Ann. Rev. Nucl. Sci.* **34**, 461 (1984); T. W. Kirkman and C. K. Zachos, *Phys. Rev.* D **24**, 999 (1981).
6. M. Turner, *Phys. Lett.* B **115**, 95 (1982); P. H. Eberhard, *et al.*, *Phys. Rev.* D **4**, 3260 (1971); R. R. Ross, *et al.*, *Phys. Rev.* D **8**, 698 (1973); P. B. Price, *et al.*, *Phys. Rev. Lett.* **52**, 1264 (1984); B. Cabrera, *Phys. Rev. Lett.* **48**, 1378 (1982); **51**, 1933 (1983); **55**, 25 (1985); H. Jeon and M. J. Longo, *Phys. Rev. Lett.* **75**, 1443 (1995), Erratum **76**, 159 (1996); M. Ambrosio *et al.*, *Phys. Lett.* B **406**, 249 (1997).
7. G. R. Kalbfliesch, K. A. Milton, M. G. Strauss, L. Gamberg, E. H. Smith, W. Luo, *"Improved Experimental Limits on the Production of Magnetic Monopoles"*; OKHEP-00-04, to be submitted to *Phys. Rev. Lett.* .
8. T. Gentile *et al.*, *Phys. Rev.* D **35**, 1081 (1987); P. B. Price, J, Guiru, and K. Kinoshita, *Phys. Rev. Lett.* **65** 149, (1990); K. Kinoshita *et al.*,

Phys. Rev. Lett. **60**, 1610 (1988); J. L. Pinfold *et al.*, *Phys. Lett.* B **316**, 407 (1993).
9. L. Gamberg, G. R. Kalbfleisch, and K. A. Milton, "Difficulties with Photonic Searches for Magnetic Monopoles," preprint OKHEP–98–02 (hep-ph/9805365); "Direct and Indirect Searches for Low Mass Magnetic Monopoles," preprint OKHEP–99–04 (hep-ph/9906526), contribution to Kurt Haller's *Festschrift.*
10. S. Weinberg, *Phys. Rev.* **138**, B988 (1965); A. Rabl, *Phys. Rev.* **179**, 1363 (1969).
11. L. Gamberg and K. A. Milton, *Phys. Rev.* D **61**, 075013 (2000)
12. R. A. Brandt, F. Neri and D. Zwanziger, *Phys. Rev. Lett.* **40**, 147 (1978); *Phys. Rev.* D **19**, 1153 (1979).
13. L. F. Urrutia, *Phys. Rev.* D **18**, 3031 (1978).
14. J. Schwinger, *Phys. Rev.* **152**, 1219 (1966).
15. J. Schwinger, K. A. Milton, W.-y. Tsai, L.L. DeRaad, Jr. and D. Clark, *Ann. Phys. (NY)* **101**, 451 (1976).
16. J. Schwinger, *Phys. Rev.* **82**, 664 (1951).
17. O. Nachtmann, *Ann. Phys. (NY)* **209**, 436 (1991);
18. F. Bloch and A. Nordsieck, *Phys. Rev.* **52**, 54 (1937).
19. G. W. Erickson and H. M. Fried, *J. Math. Phys.* **6**, 414 (1965); *Phys. Rev.* D **3**, 2010 (1971); M. Levy and J. Sucher, *Phys. Rev.* **186**, 1656 (1969); *Phys. Rev.* D **2**, 1716 (1970); H. M. Fried, *Functional Methods and Eikonal Models*, Editions Frontières, Gif-sur-Yvette Cedex, 1990.
20. H. Abarbanel and C. Itzykson, *Phys. Rev. Lett.* **23**, 53 (1969)
21. H.M. Fried, T.K. Gaisser, *Phys. Rev.* **179**, 1491 (1969); T.K. Gaisser, *Phys. Rev.* D **2**, 1337 (1971).
22. G. P. Korchemsky, *Phys. Lett.* **325 B**, 459 (1994); I. A. Korchemskya, and G. P. Korchemsky, *Nucl. Phys.* **B437**, 127 (1994).
23. G. C. Gellas, A. I. Karanikas, and C. N. Ktorides, *Phys. Rev.* D **57**, 3763 (1998).
24. H.M. Fried, private communication.